

Third Semester B. Sc. Examination
STATISTICS
Paper – I
(Statistical Methods)

Time : Three Hours] [Max. Marks : 50

N. B. : All the five questions are compulsory and carry equal marks.

1. (A) Define

- (i) Joint p. d. f. of two random variables X and Y.
- (ii) Marginal densities of X and Y.
- (iii) Covariance between two random variables X and Y.
- (iv) Stochastic independence of random variables X and Y.

The fraction X of male runners and the fraction Y of female runners who compete in marathon races is described by the joint density function

$$f(x, y) = \begin{cases} 8xy, & 0 < x < 1, 0 < y < x \\ 0, & \text{elsewhere} \end{cases}$$

Find (i) marginal densities of random variables X and Y.

- (ii) $E(X)$, $E(Y)$, $E(XY)$
- (iii) $\text{Cov}(X, Y)$
- (iv) Check the independence of X and Y. 10

OR

(E) Let $f(x, y)$ be joint p. d. f. of r. Vs. X and Y and $M(t_1, t_2)$ be mgf. of the joint probability distribution.

Let $h(x)$ and $g(y)$ be functions of x alone and y alone respectively. Show that the r. Vs. X and Y are independently distributed iff

- (i) $b(x, y) = h(x) \cdot g(y)$
- (ii) $M(t_1, t_2) = M(t_1, 0) \cdot M(0, t_2)$.

Let X_{t_1} and X_{t_2} be two independent random variables resulting from two throws of an unbiased die. Find the p. m. f. of $y = X_{t_1} + X_{t_2}$.

Also find $E(Y)$ and $V(Y)$. 10

2. (A) Derive the p. m. f. of trinomial distribution. According to a genetics theory, a certain cross guinea pigs will result in red, black and white offsprings in the ratio 8:4:4. Find the probability that among 8 offsprings 5 will be red, 2 black and 1 white. 10

OR

(E) If the random variables X and Y follow bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ and ρ , then find marginal p. d. f. of r. v. X. Also find conditional density of Y given X. Hence state conditional mean and conditional variance of Y given x. 10

3. (A) Define a random sample. Write the steps for drawing a random sample of size 10 from Binomial population with parameters σ and 0.3.
 (B) Let X be a continuous random variable with probability distribution

$$f(x) = \begin{cases} x/12, & 1 < x < 5 \\ 0, & \text{elsewhere} \end{cases}$$

find the probability distribution of the random variable $y = 2x - 3$.

(C) Let X_1 and X_2 be random variables with joint probability distribution

$$p(x_1, x_2) = \begin{cases} \frac{x_1 x_2}{18}, & x_1 = 1, 2; x_2 = 1, 2, 3 \\ 0, & \text{elsewhere} \end{cases}$$

Find the probability distribution of $Y = X_1 \cdot X_2$.

(D) The hospital period, in days, for patients following treatment for a certain type of kidney disorder is a random variable $Y = X + 4$, where X has the density function

$$f(x) = \begin{cases} \frac{32}{(x+4)^3}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find (i) the p. d. f. of Y

(ii) the probability that the hospital period for a patient following the treatment will exceed 8 days.

$$2\frac{1}{2} \times 4 = 10$$

OR

(E) X and Y are independent Gamma variates with parameters (α, λ) and (β, λ) respectively. Show that $V = X + Y$ follows Gamma distribution with parameters $(\alpha + \beta, \lambda)$ and

$$V = \frac{X}{X+Y}$$

follows Beta distribution with parameters (α, β) .

(F) If $X \sim N(\mu, \sigma^2)$ then show that $Y = a + bX, b \neq 0$ is also normally distributed with mean $a + b\mu$ and variance $b^2 \sigma^2$. Also if a random sample of size n is taken from $N(\mu, \sigma^2)$ then find the distribution of sample mean \bar{X} . 5 + 5

4. (A) Define Chi – square statistic. State the p. d. f. of Chi – square distribution. Find its m. g. f. and hence find mean and variance of this distribution. Also find mode of Chi – square distribution. 10

OR

(B) Define Fisher's t statistic. Derive p. d. f. of t distribution. Show that mean of t distribution is zero. Find its variance. 10

5. Solve any **ten** of the following :—

(A) State additive property of Chi – square distribution.
 (B) State p. d. f. of F distribution.
 (C) If X has t distribution with n d. f., derive the distribution of χ^2 .

(D) Define correlation coefficient and state its limits.

(E) Define conditional mean of a discrete random variable X given $Y = y$.

(F) If the pair of random variables (X, Y) follows Bivariate normal distribution with the parameters $\mu_x = 10, \sigma_x^2 = 9, \mu_y = 12, \sigma_y^2 = 16$ $\rho = 0.6$. Find $E(Y/x = 2)$

(G) State m. g. f. of trinomial distribution.

(H) If the joint p. m. f. of random variables X and Y is

$$f(x, y) = \frac{x+y}{21}, x = 1, 2, 3 \text{ and } y = 1, 2$$

find marginal distribution of X .

(I) If r . Vs. X and Y follow Bivariate normal distribution with $\rho = 0$ show that X and Y are independent.

(J) Define a statistic.

(K) If $X_1 \sim B(n_1, p)$ and $X_2 \sim B(n_2, p)$, state the probability distribution of $X_1 + X_2$.

(L) Let X have a p. m. f.

$$f(x) = \frac{1}{4}, x = 0, 1, 2, 3$$

Find the p. m. f. of $y = 3x$. $1 \times 10 = 10$